Thought Question

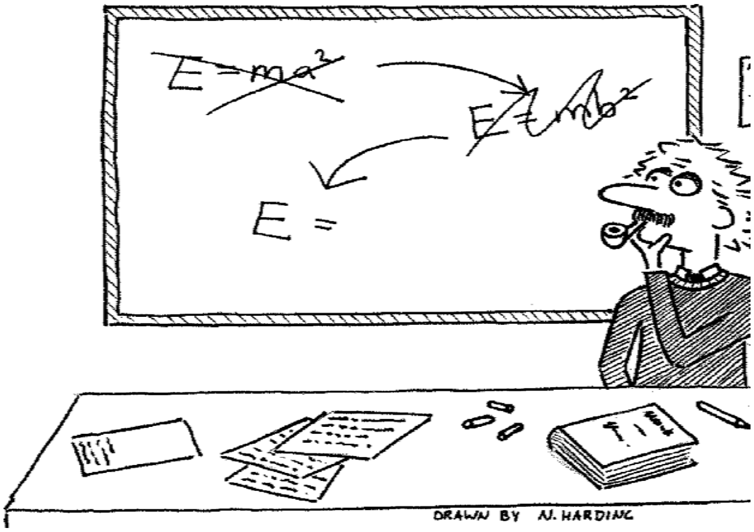
In the missionary and cannibals problem, it is generally the case that two creatures are brought over and one is then brought back, yielding a net increase in creatures on the "goal" side. In one critical step, however, **two** creatures are brought back in the boat. For each of the following search strategies, say whether a solution to this problem is:

(1) guaranteed, (2) possible but not guaranteed,

(3) impossible

a. algorithmic
b. hill climbing (simple search)
c. working backwards
d. means-ends

Problem Solving II



~~E = ma²~~
~~E = mb²~~
E =
DRAWN BY N. HARDING

Well-defined Problems

- Last time, talked about well-defined problems:
  - clear initial conditions
  - a well-specified goal state
  - a set of known operators (things you can and cannot do)
- Solving such problems involves figuring out how to best apply operators to get to goal state.

Ill-defined Problems

- Many other problems are poorly defined.
- May know initial conditions and goal (though not always clearly specified).
- But, often do **not** know what the operators are.
  - Have never seen such a problem before.
  - Don't know the rules/possibilities.
  - Have a sense of "what should I do?" rather than "which should I do?"

Example

A small country was ruled from a strong fortress by a dictator. The fortress was situated in the middle of the country, surrounded by farms and villages. Many roads led to the fortress through the countryside. A rebel general vowed to capture the fortress. The general knew that an attack by his entire army would capture the fortress. However, the general also knew that the dictator had placed mines on each of the roads. The mines were set so that a small set of people could pass over them safely, but any large force would detonate the mines. How could the general capture the fortress?

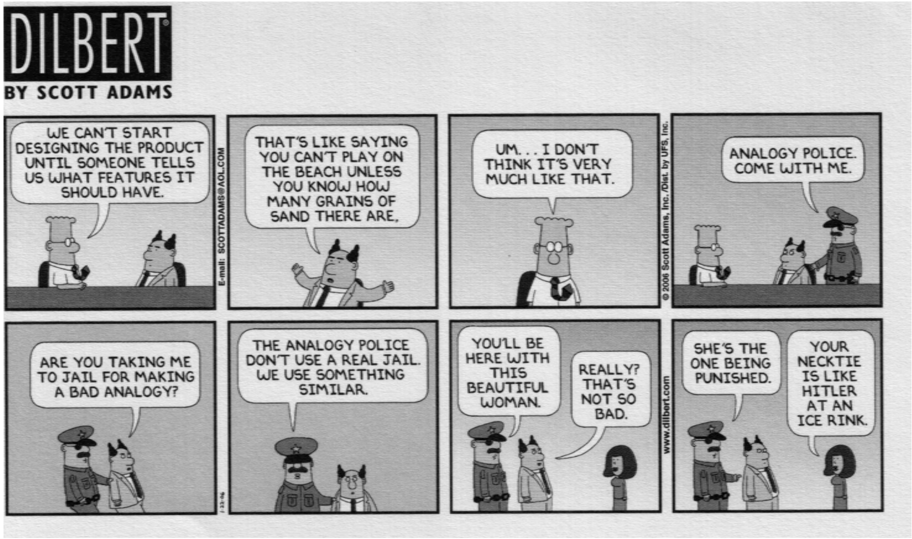
Success?

- How many have clear answer?
  - generally 10% of people
  - if still don’t know, wait a bit more

Key Factors

- Analogy
- Representation
- Incubation

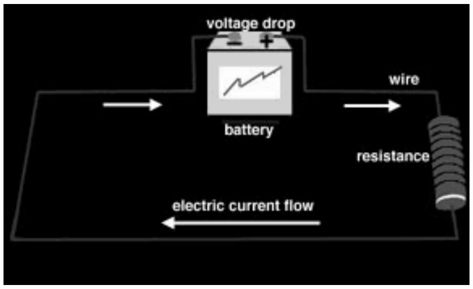
Analogy


DILBERT
BY SCOTT ADAMS
WE CAN'T START DESIGNING THE PRODUCT UNTIL SOMEONE TELLS US WHAT FEATURES IT SHOULD HAVE.
E-mail: SCOTTADAMS@AOL.COM
THAT'S LIKE SAYING YOU CAN'T PLAY ON THE BEACH UNLESS YOU KNOW HOW MANY GRAINS OF SAND THERE ARE.
UM... I DON'T THINK IT'S VERY MUCH LIKE THAT.
© 2006 Scott Adams, Inc./Dist. by UFS, Inc.
ANALOGY POLICE. COME WITH ME.
ARE YOU TAKING ME TO JAIL FOR MAKING A BAD ANALOGY?
THE ANALOGY POLICE DON'T USE A REAL JAIL. WE USE SOMETHING SIMILAR.
YOU'LL BE HERE WITH THIS BEAUTIFUL WOMAN.
REALLY? THAT'S NOT SO BAD.
www.dilbert.com
SHE'S THE ONE BEING PUNISHED.
YOUR NECKTIE IS LIKE HITLER AT AN ICE RINK.

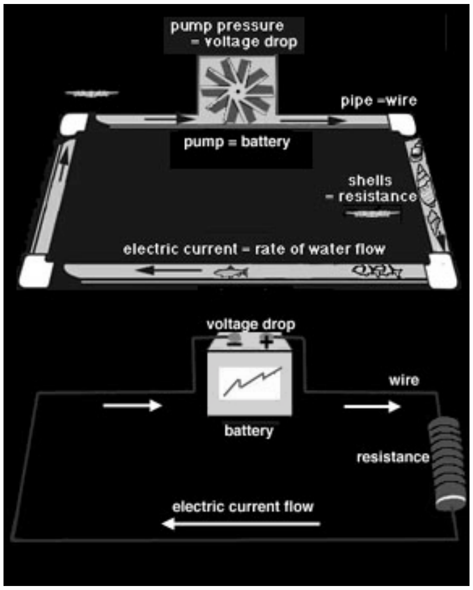
Definitions

- Problem solving by analogy involves using the solution to a different, but familiar, problem to solve a new problem.
  - **Source:** domain you have knowledge about
  - **Target:** domain you are trying to think about
  - **Mappings:** correspondences between domains

Analogy Example

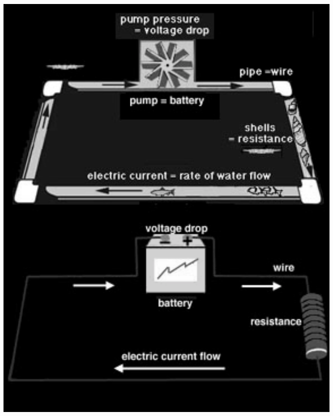


Electricity = Water Analogy



Using analogy

- ➢ Attributes and relations.
  - – Attributes:
    - has battery with two terminals
    - has pump with motor driven water wheel
  - – Relations:
    - more voltage leads to more current
    - more pressure leads to more flow
- ➢ Relations are most important.
- ➢ Need to find source domain with similar set of relations.



Another example problem

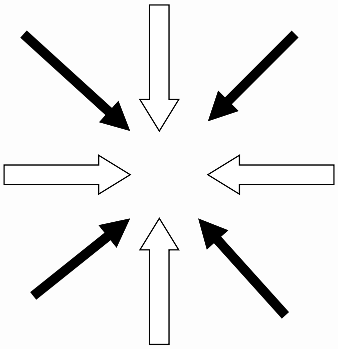
Suppose you are a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is destroyed the patient will die. There is a kind of laser that can be used to destroy the tumor. If the rays from the laser reach the tumor all at once at a sufficiently high intensity, the tumor will be destroyed. Unfortunately, at this intensity the healthy tissue that the rays pass through on the way to the tumor will also be destroyed. If a less intense setting of the laser is used, the rays are harmless to healthy tissue, but they will not affect the tumor either. What type of procedure might be used to destroy the tumor and, at the same time, avoid destroying the healthy tissue?

Another example problem

To solve this problem, the doctor devised a simple plan. She used multiple lasers to deliver low intensity rays to the tumor from different directions. Because the rays were all low intensity when passing through healthy tissue, the healthy tissue was saved. However, the tumor received a high intensity dose of the rays because many of them converged on it. Therefore, the tumor was destroyed.

Think back to the military problem

- How many have a solution now?
  - after hearing tumor story
  - **and given hint to use it**
  - generally 75%



Correspondences

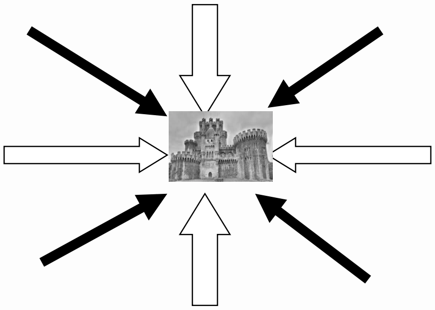
Military Problem	***Tumor Problem***
Goal: use army to capture fortress	**Goal**: use rays to destroy tumor
Resources: large army	**Resources**: powerful rays
Operators: divide army, move army, attack w/army	**Operators**: adjust ray intensity, move ray source, administer rays
Constraints: unable to safely send entire army along one road	**Constraints**: unable to safely use intense rays from one direction
Solution: send small groups along multiple roads	**Solution:** use low-intensity rays from many directions
Outcome: fortress captured by army	**Outcome**: tumor destroyed by rays

Going beyond correspondences

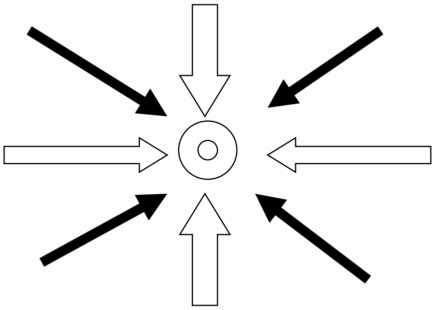
- Once you figure out correspondences, you can generate a more general form of the problem
- Called a schema

Schema: How are these the same?

- Military problem



- Tumor problem



- Schema: general form of problem. Allows you to make the analogy (small attacks from different angles).

Generalization to schema

Schema: a general representation of the critical features of the analogy

Convergence Schema

- **Goal** – use force to overcome a central target
- **Resources** – sufficiently great force
- **Operators** – change force intensity, move source of force, apply force
- **Constraints** – can't safely apply full force along one path
- **Solution** – apply weak forces along multiple paths
- **Outcome** – central target overcome by force

Problems

- BUT, people often have difficulty drawing analogies, even when you know they have the knowledge they need.
- Illustration: people are given the military problem and then the tumor problem, but are not **told** to relate them.
  - performance on the military problem only improves from 10% to 30%
  - people need to be *directed* to see similarity

Why so hard?

- Surface similarity vs. structural similarity
- Surface similarity refers to objects and ideas in the problem domain (the attributes)
  - lasers, armies
  - tumors, dictators
- Structural similarity refers to relations and types of schemas
  - convergence

Why so hard?

- People often seem driven by surface similarity.
  - not so good transfer between tumor problem and military problem (structural similarity)
  - much better transfer between tumor problem and problem in which a lightbulb must be fixed with lasers (structural **and** surface similarity)

Testing use of similarity

- Take people who have seen tumor problem and solution. Give a version of the lightbulb problem.
- Vary surface similarity – vary instrument
  - laser vs. ultrasound
- Vary structural similarity – vary constraint
  - fragile glass vs. insufficient intensity

Success Rates: no hint provided

Best performance: most similar to tumor problem

So-so performance: structure is same but surface cue different

	Surface Cue		
Structural	*Laser*	*Ultrasound*	*Mean*
Fragile-glass	69%	38%	54%
Insufficient-intensity	33%	13%	23%
	51%	26%	

So-so performance: surface cue is same but structure is different

Worst performance: least similar to tumor problem

Success Rates: hint IS provided

	Surface Cue		
Structural	*Laser*	*Ultrasound*	*Mean*
Fragile-glass	75%	81%	78%
Insufficient-intensity	60%	47%	54%
	68%	64%	

Structural similarity always matters

With hint, no difference between surface cue type

Implications

- Both surface & structural similarity help, but in different ways.
- Surface similarity helps people think to try using their knowledge of the analogous problem.
- BUT, structural similarity more important for drawing appropriate analogical inferences.

- Brainstorming: may help break people away from surface similarity; more likely to find structurally similar source.

Aids to analogy

People seem to use analogous problems more effectively when they:

- are explicitly told to compare two problems that initially seem unrelated
- are shown *several* structurally similar problems before trying the target problem
- actually try to solve the source problem rather than just hearing the solution

---

Representation

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"My team has created a very innovative solution, but we're still looking for a problem to go with it."

Sensitivity to Representation

- People are sensitive to how a problem is represented ("framed").
- Some representations lead to more solutions or faster solutions.

Number scrabble

1 2 3 4 5

6 7 8 9

- Two people play, picking cards in turn.
- Goal is to be the first to get three cards that add up to 15.

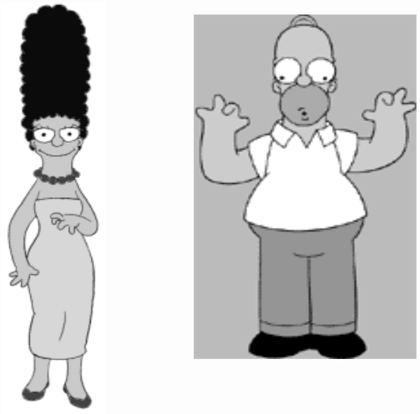
3

6 7 8

9

1

5



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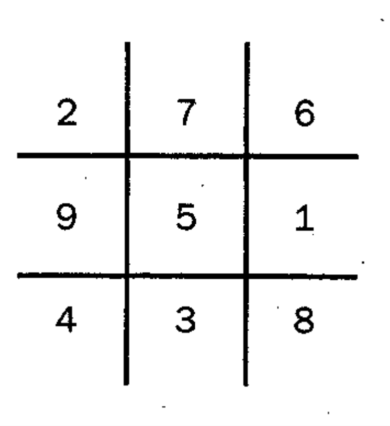
A Different Representation

- ➢ People find number scrabble difficult at first.
- ➢ Much easier with a different representation:



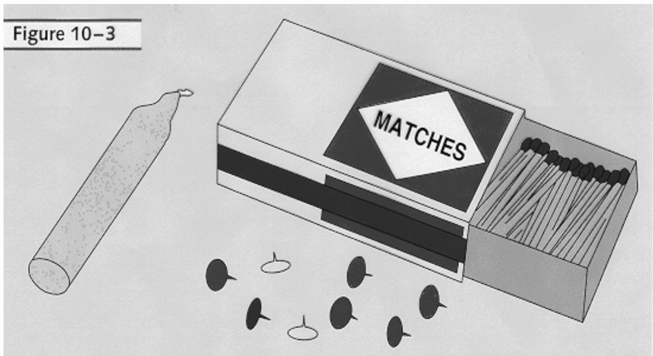
2	7	6
9	5	1
4	3	8

Centrality of Representation

- Representation is crucial.
- Representation:
  - guides search for related problems
  - helps focus on appropriate attributes/relations
  - determines how efficiently operators are applied
  - ultimately, shapes whether problem is solved

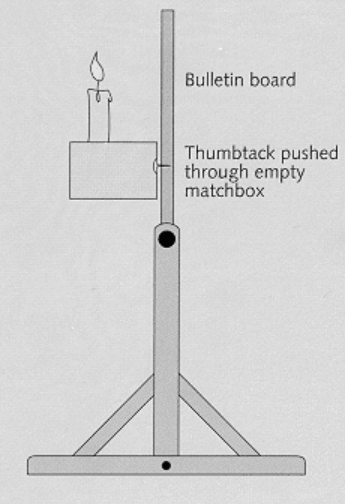
Example

Figure 10–3



You have a candle, some thumb tacks and a box of matches. Your task is to attach the candle to a bulletin board so that it can provide light to the room.

Example



- ➢ Solution:
  - – use match box
  - – attach to wall with tack
  - – use as base for candle
- ➢ People often fail to think of using the box in a novel way.
- ➢ Functional fixedness

Functional fixedness

- ➢ Functional fixedness: difficulty thinking about familiar objects in novel ways.
- ➢ Problem solving often requires considering the properties of objects in more depth.
  - – box as container
  - – box as support
  - – box as source of cardboard material
  - – etc.

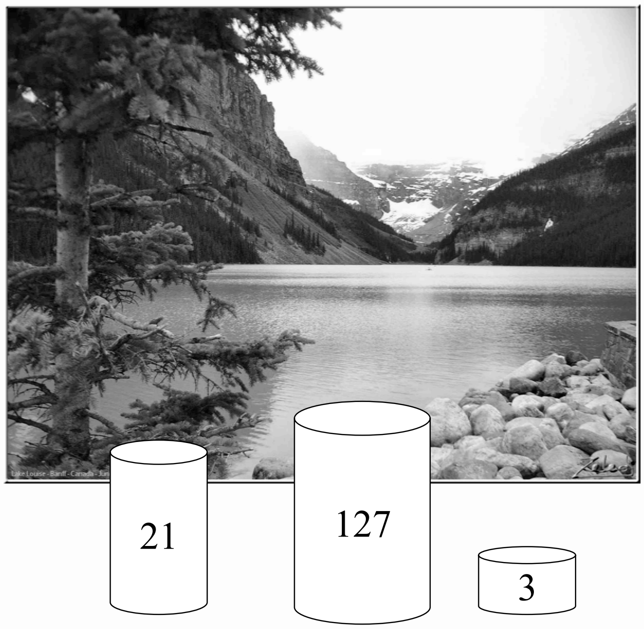
Another example

You are in a room with three light switches, all in the "off" position. These switches control three incandescent light bulbs in the next room. The door to that room is closed and sealed and there are no windows, so there is no way to see the light bulbs from this room. Your task is to determine the mapping from the three switches to the three light bulbs. You are allowed to turn the switches on and off as many times as you like. However, you may only walk into the room containing the light bulbs exactly once and once you walk in, you may no longer use the switches. How can you establish the mapping?

Another example

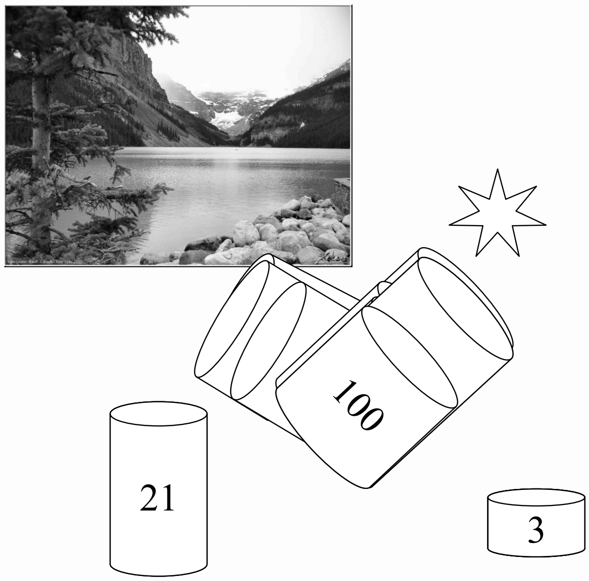
- ➢Consider properties of light bulbs:
  - – give light
  - – **get hot**
- ➢SOLUTION:
  - – leave one switch off
  - – turn one switch on
  - – turn third switch on for some time and then off
  - – When enter room, should find one lightbulb on, one off and cold, one off and hot

Example: Water Jug problem



Fill to 100 cups

Example: Water Jug problem



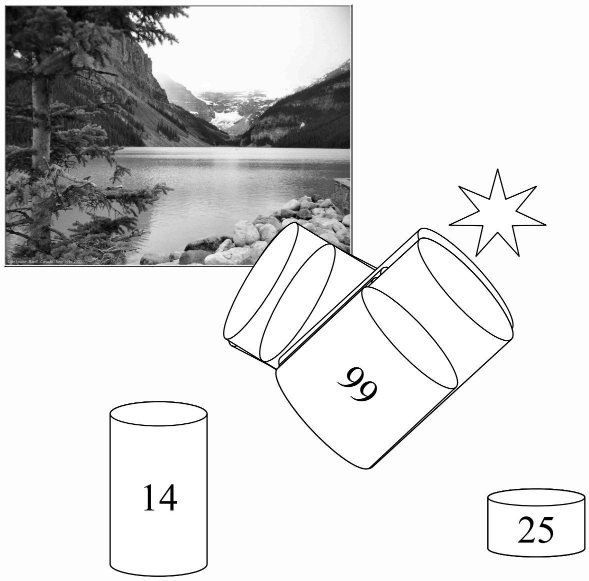
Fill to 100 cups

Example: Water Jug problem

Luchin's 1939 water jug problems

Prob	JugA	JugB	JugC	Desired
1	21	127	3	100
2	14	163	25	99

Example: Water Jug problem #2



Fill to 99 cups

Set Effects

Luchin's 1939 water jug problems

Prob	JugA	JugB	JugC	Desired
1	21	127	3	100
2	14	163	25	99
3	18	43	10	5
4	9	42	6	21
5	20	59	4	31

Simplest Solution: B – 2C - A

Set Effects

Prob	JugA	JugB	JugC	Desired
6	23	49	3	20
7	15	39	3	18

83% use B – 2C – A on # 6 and #7;
but can do (A-C) for #6 and (A+C) for #7!

8	28	76	3	25

64% can't solve #8 because B – 2C – A won't work;
but just need (A -C)!

Problems with problem solving

- People need to learn to:
  - see beyond the surface characteristics of the problem
  - see beyond the “obvious” characteristics of familiar objects
  - use different strategies for similar problems when appropriate
  - **find the appropriate representation for the problem and its elements**

Representational Change

- Overcoming functional fixedness and set effects requires changing one’s representation of the problem: restructuring.
- When this happens, sometimes experience feeling of insight -- “A-HA!”
  - Arises when people do not have a clear sense of their proximity to the goal.
  - Yet, probably isn’t a sudden change . . . instead, a gradual relaxation of constraints.

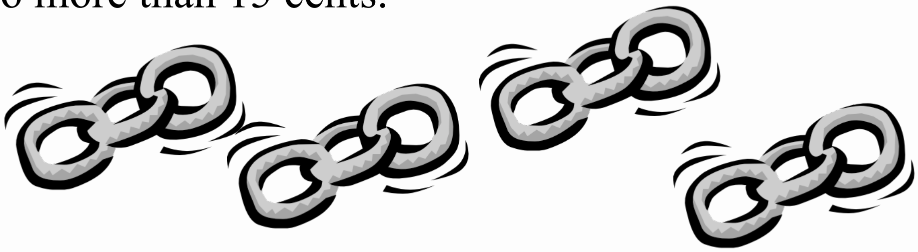
Incubation

Waiting . . .

- Incubation: time away from problem
- Idea that sometimes need to "stop thinking about" a problem in order to find solution.
- Scientifically valid?

Example

You are given four separate pieces of chain that are each three links in length. It costs 2 cents to open a link and 3 cents to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15 cents.



Three groups

- Group A: works for 30 minutes
- Group B: works for 15 minutes, takes a 30 minute break (doing something else), works another 15 minutes
- Group C: works for 15 minutes, takes a 4 hour break (doing something else), works another 15 minutes.

Three groups

- All Groups spend 30 minutes actively working on the problem.
- Individuals in Groups B and C, who take breaks, do **not** come back from breaks with problem already solved.

Three groups

- 55% of people in Group A solve problem.
- 64% of people in Group B (30 minute incubation time) solve problem.
- 85% of people in Group C (4 hour incubation time) solve problem.

Solution

You are given four separate pieces of chain that are each three links in length. It costs 2 cents to open a link and 3 cents to close a link. All links are closed at the beginning of the problem. Your goal is to join all 12 links of chain into a single circle at a cost of no more than 15 cents.

Open all three links on one chain (6 cents). Use to connect other three (9 cents).

Why does incubation work?

- Allows people to break away from set effects created by trying to solve problem with incorrect approaches.
- Without incubation period, people tend to try the "same" thing over and over.

Summary

- Problem solving involves a search for relevant knowledge from memory.
- Hard problem because we have a lot of knowledge .
- How do we figure out what is relevant?

Summary

- Humans tend to start what is readily available:
  - surface features of the problem representation
  - obvious characteristics of objects
  - recently used procedures
- But when this fails, need ways of bringing new information in:
  - brainstorming
  - incubation

What can we do to improve problem-solving?

- Grant & Spivey (2003): monitored eye movements in tumor problem
  - successful participants looked more at the "skin" of the person

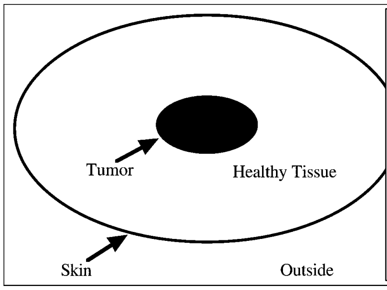
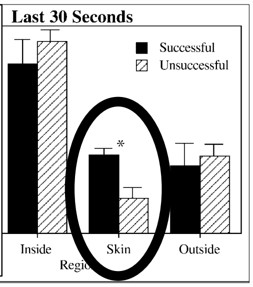


Fig. 1. The diagram that participants viewed while solving Duncker's (1945) radiation problem. The labels of diagram features were not shown; the features were verbally explained.

everybody looks at the inside… but that's not what helps! it's looking at the skin!



What can we do to improve problem-solving?

- Can we improve performance by drawing attention to key area (the "skin")?
  - YES! Success increased when "skin" pulsed
  - Success decreased when "tumor" pulsed

Shows that attention to the right details of a problem helps!

Best performance when "skin" pulsed

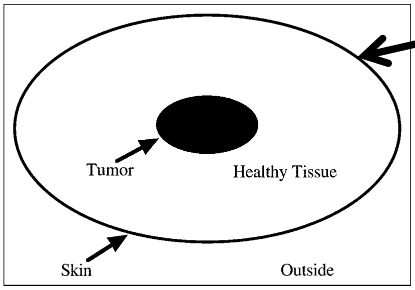
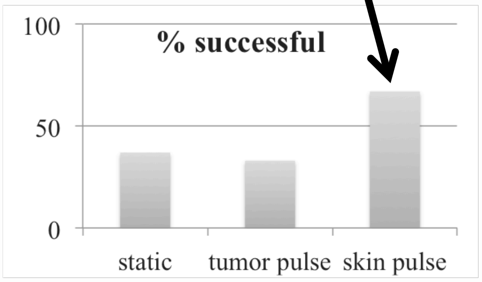


Fig. 1. The diagram that participants viewed while solving Duncker's (1945) radiation problem. The labels of diagram features were not shown; the features were verbally explained.

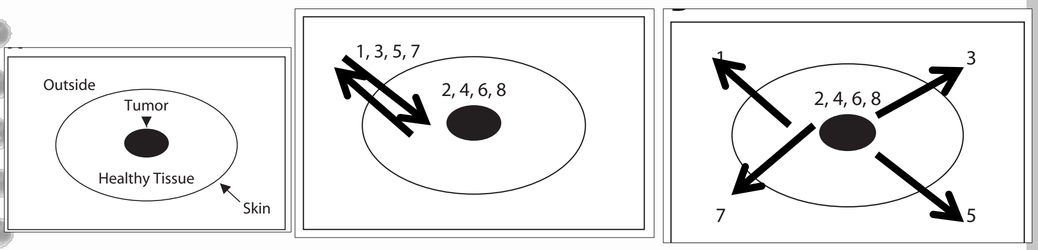


What can we do to improve problem-solving?

- Thomas & Lleras (2007): Why is looking at the skin helpful? Two hypotheses:
- Embodiment hypothesis:
  - eyes move in the same direction that the lasers would move
- Eye movement hypothesis:
  - just moving your eyes between skin and tumor is enough (looking at features is all that matters; moving in same way as lasers is irrelevant)

Tumor problem: Why is looking at the skin helpful?

- Task is to solve the problem, and also move eyes to digits presented on screen.



eye movement
19% solve rate

embodiment
50% solve rate

- Consistent with embodiment: when eyes move like the lasers you do better.

Thought Question

An experimenter asks two groups of subjects to solve both the Missionary and Cannibals problem and the tumor problem. Each group works on each problem for a total of 30 minutes. Group 1 works continuously on each problem, while Group 2 takes a fifteen minute break (doing another task) in the middle of working on each. The experimenter obtains the following results:

	M&C	Tumor
Group 1	80%	10%
Group 2	77%	50%

What might we conclude from these results about incubation and problem solving?